

1. A block of mass  $m$  on a horizontal surface is attached to a spring having spring constant  $k$ . We pull the mass a distance  $x = A$  to the right of  $x = 0$ , which is the equilibrium position, and let it go. How far to the left of the equilibrium position  $x = 0$  will it reach, assuming that the kinetic coefficient of friction between the mass and the surface is  $\mu_k$ ?

- (A)  $\frac{-A}{2}$   
 (B)  $-2A + \frac{2\mu_k mg}{k}$   
 (C)  $\frac{\mu_k mg}{k}$   
 (D)  $-2A$   
 (E)  $-A + \frac{2\mu_k mg}{k}$

2. A completely inelastic collision occurs between a mass  $m_1$  moving upward with velocity  $v_1$  and a mass  $m_2$  moving downward at velocity  $v_2$ . How high does the combined mass rise above the point of collision? The collision occurs close to the surface of earth; ignore air resistance.

- (A)  $\frac{1}{2g} \left( \frac{m_1 v_1 - m_2 v_2}{m_1 + m_2} \right)$   
 (B)  $\frac{m_1 v_1}{g}$   
 (C)  $\frac{1}{g} \left( \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \right)$   
 (D)  $\frac{1}{g} \left( \frac{m_1 v_1}{m_1 + m_2} \right)$   
 (E)  $\frac{1}{g} \left( \frac{m_2 v_2}{m_1 + m_2} \right)$

3. A girl of mass  $M$  stands on the rim of a frictionless merry-go-round of radius  $R$  and rotational inertia  $I$  that is not moving. She throws a rock of mass  $m$  horizontally in a direction that is tangent to the outer edge of the merry-go-round. The speed of the rock, relative to the ground, is  $v$ . Afterwards what is the angular speed of the merry-go-round?

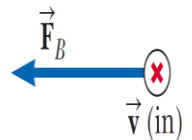
- (A)  $\frac{mvR}{I}$   
 (B)  $\frac{MvR}{mR^2 + I}$   
 (C)  $\frac{mvR}{MR^2 + I}$   
 (D)  $\frac{mv}{MR}$   
 (E)  $\frac{v}{R}$

4. A spherical charge distribution has charge density  $\rho$  that is constant from  $r = 0$  to  $r = R$  and zero beyond. What is the electric field for all values of  $r$  less than  $R$ ?

- A)  $E = \frac{\rho r}{3\epsilon_0}$   
 B)  $E = \frac{\rho}{3\epsilon_0 r}$   
 C)  $E = \frac{\rho r^2}{3\epsilon_0}$   
 D)  $E = \frac{\rho}{3\epsilon_0 r^2}$   
 E)  $E = \frac{\rho}{3\epsilon_0}$

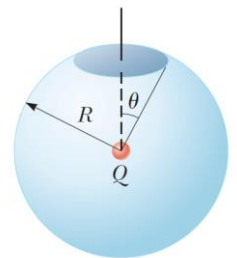
5. Find the direction of the magnetic field acting on a positively charged particle moving in the direction which is into the page, as shown in the figure. The direction of the magnetic force acting on it is:

- (A) Into the page  
 (B) To the right  
 (C) To the left  
 (D) Towards the bottom of the page  
 (E) Towards the top of the page



6. A sphere of radius  $R$  surrounds a particle with charge  $Q$  located at its center as shown in the figure. The electric flux through a circular cap of half-angle  $\theta$  is

- A)  $Q/\epsilon_0$   
 B)  $Q/2\epsilon_0$   
 C)  $\left(\frac{Q}{\epsilon_0}\right) \cos\theta$   
 D)  $\left(\frac{Q}{2\epsilon_0}\right) (1 - \cos\theta)$   
 E)  $\left(\frac{Q}{\epsilon_0}\right) (\cos\theta - 1)$



7. The expression for free energy of a system, connected to a heat bath at temperature  $T$  and with two non-degenerate states, one at energy 0 and other at  $\varepsilon$ , is

A)  $F = -kT \log(1 + e^{-\varepsilon/kT})$

B)  $F = -kT \log(1 - e^{-\varepsilon/kT})$

(C)  $F = -kT \log(1 - e^{\varepsilon/kT})$

(D)  $F = -kT \log(1 + e^{\varepsilon/kT})$

(E)  $F = kT \log(1 + e^{-\varepsilon/kT})$

8. An adiabatic expansion of an ideal gas is described by the expression  $PV^\gamma = C$ , where  $\gamma$  and  $C$  are constants. The work done by the gas in expanding from  $(P_i, V_i)$  to  $(P_f, V_f)$  is

(A)  $P_f V_f$

(B)  $\frac{P_i + P_f}{2} (V_f - V_i)$

(C)  $\frac{(P_f V_f - P_i V_i)}{1 - \gamma}$

(D)  $\frac{(P_f V_f - P_i V_i)}{1 + \gamma}$

(E)  $\frac{P_f (V_f^{1-\gamma} - V_i^{1-\gamma})}{1 + \gamma}$

9. If we consider the spin of an electron, the maximal occupation number of an energy level in an ideal electron gas is
- (A) Zero
  - (B) One
  - (C) Two
  - (D) Three
  - (E) Four
10. For an isolated quantum mechanical system originally in the state  $\Psi = a\phi_n + b\phi_m$  which on measurement is found to be in state  $\phi_m$  at time  $t$ . What state will it be found in if a second experimental measurement is made at a time  $t'$  later than  $t$ ?
- A)  $\phi_m$
  - B)  $\phi_n$
  - C)  $a\phi_n + b\phi_m$
  - D) Both  $\phi_n$  and  $\phi_m$  will have equal chance to be found
  - E) None of the above
11. At a given instant of time, a rigid rotator is in the state  $\psi(\vartheta, \phi) = \sqrt{3/4\pi} \sin \vartheta \sin \phi$ , where  $\vartheta$  is the polar angle relative to the z-axis and  $\phi$  is the azimuthal angle. Measurement will find which of the following possible values of the z-component of the angular momentum  $L_z$ ?
- (A) 0
  - (B)  $\hbar/2, -\hbar/2$
  - (C)  $\hbar, -\hbar$
  - (D)  $2\hbar, -2\hbar$
  - (E)  $\hbar, 0, -\hbar$
12. What is the expectation value of position for a free particle of mass  $m$  moving in one dimension having momentum  $p_x$ ?
- (A) 0
  - (B)  $x$
  - (C)  $2x$
  - (D)  $\infty$
  - (E)  $x/2$

13. A three-dimensional Harmonic Oscillator is in state  $E_n = \frac{9\hbar\omega}{2}$ , what is the degeneracy of this state?
- (A) 0  
(B) 3  
(C) 5  
(D) 7  
(E) 9
14. Consider a physical system associated with a three dimensional state space. An orthonormal basis for this system is given by  $|1\rangle, |2\rangle, |3\rangle$  and the Hamiltonian of this system in this basis is given by the matrix:  $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ . What are the possible results if a measurement of energy is made, in appropriate units?
- (A) 1,2,3  
(B) 2,2,2  
(C) 1,1,1  
(D) 3 and 3  
(E) 1 and 3
15. You must have seen that one can obtain the classical Maxwell-Boltzmann distribution from the quantum Fermi-Dirac and Bose-Einstein distributions:  $\frac{1}{e^{\beta(E_i-\mu)} \pm 1}$  where  $\beta = \frac{1}{kT}$ . This is obtained when
- (A)  $e^{\beta(E_i-\mu)} \gg 1$   
(B)  $e^{\beta(E_i-\mu)} \ll 1$   
(C)  $e^{\beta(E_i-\mu)} = 1$   
(D)  $e^{\beta E_i} = 1$   
(E)  $e^{\beta\mu} \gg 1$

16. Consider an ideal gas having  $N$  particles in a container attached to a piston. The system is at temperature  $T$ . We let the gas expand isothermally in a quasi-static process from initial volume  $V_i$  to final volume  $V_f$ . If the workdone by the gas during the process is

$NkT \ln\left(\frac{V_f}{V_i}\right)$ , what is the heat input?

- A)  $kT$
- B)  $NkT$
- C)  $NkT \ln\left(\frac{V_f}{V_i}\right)$
- D)  $\frac{3}{2} NkT$
- E)  $\frac{1}{2} NkT$

17. The real and imaginary parts of  $z = \frac{1+i}{1-i}$  are

- (A)  $(\text{Re}(z), \text{Im}(z)) = (0,1)$
- (B)  $(\text{Re}(z), \text{Im}(z)) = (1,1)$
- (C)  $(\text{Re}(z), \text{Im}(z)) = (-1,1)$
- (D)  $(\text{Re}(z), \text{Im}(z)) = (1,-1)$
- (E)  $(\text{Re}(z), \text{Im}(z)) = (0,0)$

18. Eigenvalues of the matrix  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$  are

- A)  $(0,0)$
- B)  $(0,1)$
- C)  $(1,0)$
- D)  $(1,-1)$
- E)  $(-1,-1)$

19. Expansion of  $\sin^2 x$  about  $x = 0$  is

- A)  $x^2 - \frac{x^4}{3} + \frac{2x^6}{45} + \dots$
- B)  $x^2 + \frac{x^4}{3} - \frac{2x^6}{45} + \dots$
- C)  $x - \frac{x^3}{3} + \frac{2x^5}{15} + \dots$
- D)  $1 + x^2 - \frac{x^4}{3} + \dots$
- E)  $1 - x^2 + \frac{x^4}{3} + \dots$

20. Consider the function  $f(x, y) = \sin x + \sin y$ . Which of the following points  $(x, y)$  is the stationary point of  $f(x, y)$ ?

- A)  $(\frac{\pi}{2}, \frac{\pi}{2})$
- B)  $(0, 0)$
- C)  $(\frac{\pi}{2}, 0)$
- D)  $(0, \frac{\pi}{2})$
- E)  $(\pi, \pi)$

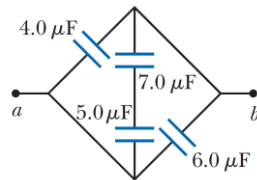
21. For  $\mathbf{v} = (y, ax, bz)$  to be a divergence-less and curl-less vector, the values of  $a$  and  $b$  are

- A)  $(a, b) = (0, 0)$
- B)  $(a, b) = (0, 1)$
- C)  $(a, b) = (1, 0)$
- D)  $(a, b) = (-1, 0)$
- E)  $(a, b) = (0, -1)$

22. A classical system with one degree of freedom has a Hamiltonian  $H(q, p) = \frac{p^2}{2m} + A(q)p + B(q)$ , where  $A$  and  $B$  are functions of the coordinate  $q$  and  $p$  is the momentum conjugate to  $q$ . The velocity  $\dot{q}$  is
- A)  $\frac{p}{m}$
  - B)  $A(q)$
  - C)  $\frac{p}{m} + B(q)$
  - D)  $B(q)$
  - E)  $\frac{p}{m} + A(q)$

23. The equivalent capacitance between points  $a$  and  $b$  in the combination of capacitors shown in Figure, is:

- A)  $12.9 \mu\text{F}$
- B)  $11.9 \mu\text{F}$
- C)  $14.9 \mu\text{F}$
- D)  $10.8 \mu\text{F}$
- E)  $9.9 \mu\text{F}$



24. The position vector of a particle moving in a circle of radius  $R$  with angular velocity  $\omega$  at time  $t$  is  $\vec{r}(t) = R(\cos(\omega t)\hat{i} + \sin(\omega t)\hat{j})$ . Its velocity at time  $t = 0$  is
- A)  $0$
  - B)  $2\omega R\hat{j}$
  - C)  $\omega R\hat{i}$
  - D)  $\omega R\hat{j}$
  - E)  $-\omega R\hat{j}$

25. Two point charges are located on the  $x$ -axis,  $q_1 = -e$  at  $x = 0$  and  $q_2 = +e$  at  $x = a$ . The work that must be done by an external force to bring a third point charge  $q_3 = +e$  to  $x = 2a$  from infinity is

- A)  $\frac{+e^2}{8\pi\epsilon_0 a}$
- B)  $\frac{+e^2}{4\pi\epsilon_0 a}$
- C)  $\frac{-e^2}{8\pi\epsilon_0 a}$
- D)  $\frac{+e^2}{a}$
- E)  $\frac{+e}{8\pi\epsilon_0 a}$