1. For a free particle moving in one dimension in state ϕ_k, where k is the wave number. What is the uncertainty in its momentum Δp?

   A) Δx
   B) Δk
   C) Infinite
   D) Zero
   E) $\hbar k$

2. If $\hat{A}$ and $\hat{B}$ are two Hermitian operators then which one of the following is a unitary operator?

   A) $i\hat{A}$
   B) $\hat{A}\hat{B}$
   C) None of these
   D) $e^{\hat{A}\hat{B}-\hat{B}\hat{A}}$
   E) $e^{\hat{A}\hat{B}+\hat{B}\hat{A}}$

3. A particle is in the state $|\Psi\rangle$. The expectation value of the operator $A = |r\rangle\langle r|$ in this state is

   A) $|\Psi(r)|^2$
   B) $\Psi(r)$
   C) r
   D) r^2
   E) None of these

4. Let us consider the work done by an external force on an object of mass m to accelerate it from a speed v to a speed $2v$ on a frictionless, horizontal surface. Is this work done

   (A) equal to the work required to accelerate the object from $v = 0$ to v
   (B) twice the work required to accelerate the object from $v = 0$ to v
   (C) three times the work required to accelerate the object from $v = 0$ to v
   (D) four times the work required to accelerate the object from 0 to v
   (E) not known without knowledge of the acceleration?

5. A car of mass *m* traveling at speed *v* crashes into the rear of a truck of mass 2*m* that is at rest and in neutral at an intersection. If the collision is perfectly inelastic, what is the speed of the combined car and truck after the collision?

   (A) *v*
   (B) *v*/2
   (C) *v*/3
   (D) 2*v*
   (E) None of these answers is correct.

6. A particle of mass *m* moves in the *xy* plane with a velocity of $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$. The angular momentum is

   (A) $\vec{L} = m(xv_y - yv_x)\hat{k}$
   (B) $\vec{L} = m(-xv_y - yv_x)\hat{k}$
   (C) $\vec{L} = m(yv_y - xv_x)\hat{\imath}$
   (D) $\vec{L} = m(xv_y - yv_x)\hat{\jmath}$
   (E) $\vec{L} = m(xv_y + yv_x)\hat{k}$

7. Suppose that each system in an ensemble is a single dice and n denotes the number of dots showing upwards. Let f be the square of dots showing upwards ($f_n = n^2$). What is the mean value of f if large number of dice is rolled?
   A) $\frac{1}{36}$
   B) $\frac{1}{6}$
   C) $\frac{71}{6}$
   D) $\frac{91}{6}$
   E) $\frac{55}{36}$

8. For a certain system at temperature T and of volume V, there is only one accessible state and it has energy $E = NkTln\left(\frac{V_0}{V}\right)$, where V_0 is a constant. The partition function for this system is
   A) $\frac{V}{V_0}$
   B) $\frac{V_0}{V}$
   C) $\left(\frac{V}{V_0}\right)^N$
   D) $ln\left(\frac{V}{V_0}\right)$
   E) $\left(\frac{V_0}{V}\right)^N$

9. For an ideal non-relativistic electron gas comprising N particles at zero temperature confined in a two-dimensional square box with side length L; the total internal energy is
   A) $\left(\frac{\hbar}{m}\right)\left(\frac{N}{L}\right)$
   B) $\left(\frac{\hbar^2}{m}\right)\left(\frac{N}{L}\right)$
   C) $\left(\frac{\pi\hbar}{m}\right)\left(\frac{N^2}{L}\right)$
   D) $\left(\frac{\pi\hbar^2}{m}\right)\left(\frac{N^2}{L^2}\right)$
   E) $\left(\frac{\pi\hbar^2}{2m}\right)\left(\frac{N^2}{L^2}\right)$

10. The magnitude of electric field intensity due to a long straight wire having a uniform line density of charge λ at a distance d perpendicular to the wire is
    A) $E = \frac{\lambda}{4\pi\epsilon_0 d^2}$
    B) $E = \frac{\lambda}{2\pi\epsilon_0 d}$
    C) $E = \frac{\lambda}{2\lambda\epsilon_0 \sqrt{d}}$
    D) $E = 0$
    E) $E = \infty$

11. Two parallel wires carrying currents in anti-parallel direction
    A) Attract each other
    B) Repel each other
    C) Neither attract nor repel each other
    D) Starts vibrating
    E) Starts rotating around each other

12. The electric potential due to a dipole varies with distance *r* as
    A) $\frac{1}{r}$
    B) $\frac{1}{r^2}$
    C) r
    D) r^2
    E) $\frac{1}{r^3}$

13. The difference between the energy of electronic states in an atom is of the order of an electron volt. At what temperature will the electronic degrees of freedom become relevant? Hint: Thermal energy at room temperature is ≈ 25meV.
    A) 1 Kelvin
    B) 10 Kelvin
    C) 100 Kelvin
    D) 1000 Kelvin
    E) 10000 Kelvin

14. Consider a spin-1/2 in a magnetic field B. The $S_z = 1/2$ state has energy $g\mu_B B/2$ and $S_z = -1/2$ has energy $-g\mu_B B/2$. Assume the spin is in contact with a heat bath at temperature T. The probability for the spin to be in the $S_z = 1/2$ state is
    A) $\frac{1}{1+e^{\beta g \mu_B B}}$ Here $\beta = \frac{1}{kT}$
    B) $\frac{1}{1+e^{-\beta g \mu_B B}}$
    C) $e^{\beta g \mu_B B}$
    D) $e^{-\beta g \mu_B B}$
    E) $2e^{\beta g \mu_B B}$

15. At $t = 0$, a particle of mass m trapped in an infinite square well of width L is in a superposition of the first excited state and the fifth excited state,

    $\Psi(x) = A(3\phi_1(x) - 2i\phi_5(x))$,

    where the $\phi_n(x)$ are orthonormal energy eigenstates with energies E_n. What should A be for $\Psi(x)$ to be normalized?
    A) $\frac{1}{\sqrt{5}}$
    B) $\frac{i}{5}$
    C) $\frac{-i}{\sqrt{12}}$
    D) $\frac{1}{13}$
    E) None of the above

16. The orbital degeneracy (excluding spin) of hydrogen atom energy levels is given, in terms of the principal quantum number n, by

    A) n − 1
    B) n
    C) n + 1
    D) 2n + 1
    E) n^2

17. For a function $f(x,y) = Axy^2$ with $x(t) = Ct^3, y(t) = Dt^2$. The derivative $\frac{\partial f}{\partial t}$ is

A) ACD

B) $4ACD^2t$

C) $5ACD^2t^6$

D) $7ACD^2t^6$

E) $7AD^2t^6$

18. A uniform ladder of length l , rests against a smooth, vertical wall (see Fig). The mass of the ladder is m, and the coefficient of static friction between the ladder and the ground is μ. The minimum angle θ_{min} at which the ladder does not slip is given by

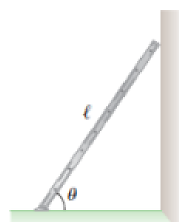
A) $\sin^{-1}(\frac{1}{2\mu})$

B) $\cos^{-1}\left(\frac{1}{2\mu}\right)$

C) $\tan^{-1}\left(\frac{1}{2\mu}\right)$

D) $\sec^{-1}\left(\frac{1}{2\mu}\right)$

E) $\cot^{-1}(\frac{1}{2\mu})$



19. A certain charge Q is divided into two parts q and $Q - q$ which are then separated by a certain distance r. What must q be in terms of Q to maximize the electrostatic repulsion between the charges?

A) Q
B) Q/2
C) 3Q/2
D) Q/4
E) 2Q

20. A particle of mass m moves under the influence of the potential $V(x) = \frac{A}{x^2} - \frac{B}{x}$ where $A, B > 0$. The frequency of small oscillations around the equilibrium point is

A) $\sqrt{\frac{B}{mA}}$

B) $\sqrt{\frac{B^4}{mA^3}}$

C) $\sqrt{\frac{B^4}{8mA^3}}$

D) $\sqrt{\frac{B^4}{2mA^3}}$

E) $\sqrt{\frac{B^4}{6mA^3}}$

21. A certain wire has a resistance R. What is the resistance of a second wire, made of the same material, that is half as long and has half the diameter?

A) R
B) 2R
C) 3R
D) R/2
E) R/3

22. The Jacobian $\left|\frac{\partial(Q,P)}{\partial(q,p)}\right|$ of the transformation, $P = p\cos q,\ Q = p\sin q$ is

A) qp
B) -1
C) $1/p$
D) 1
E) p

23. The sum of the eigenvalues of $M = \begin{pmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{pmatrix}$

A) 0
B) 8
C) 1
D) 5
E) 4

24. Evaluate the complex integral $\oint_C \frac{e^z}{z-2} dz$ around the circle $|z| = 1$

A) $2i\pi\, e$
B) $2i\pi\, e^2$
C) π
D) $-2i\pi\, e^2$
E) 0

25. Expansion of $\left(\frac{\sin x}{x}\right)^2$ about $x = 0$ is

A) $1 - \frac{x^2}{6} + \frac{x^4}{120} - \cdots$
B) $1 + \frac{x^2}{3} - \frac{2x^4}{45} - \cdots$
C) $1 - \frac{x^2}{3} + \frac{2x^4}{45} - \cdots$
D) $1 + \frac{x^3}{3} - \frac{2x^4}{45} - \cdots$
E) $1 - \frac{x^3}{3} + \frac{2x^4}{45} - \cdots$